Public Wage Bargaining, Unemployment, and Inequality *

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Abstract

In many countries, public sector nominal wages are almost identical in regions with different costs of living. In most cases this is the result of highly centralized pay systems. By developing a two-region general equilibrium model with unions and search frictions in the labour market, I study the differences in terms of unemployment, real wages, and inequality between a regional wage bargaining process and a national one in the public sector. Adopting the latter raises the real public pays in the poorer region, but at the price of higher unemployment and lower real wages in the private sector. In the richer region, unemployment decreases and private employees receive higher salaries. Finally, a national bargaining process may enhance inequality both within and between regions.

Keywords: public sector wages; unemployment; fiscal federalism
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1 Introduction

The structure of public sector wages and their interplay with the earnings in the private sector have been the subject of a vast theoretical and empirical literature (see Gregory and Borland, 1999). In this paper I focus on a related aspect that has gained less attention. In many countries, public sector salaries are very similar in nominal terms for employees of regions with different private sector productivities and costs of living. The spatial distribution of public wages is very compressed in the five largest European Union economies (Germany, France, U.K., Italy, and Spain) (see Elliot *et al.*, 2007)[1]. Albeit in a weaker form, even the United States federal government regional pays are substantially unaffected by local market conditions, while different is the case for state and local public employees (Katz and Krueger, 1991).

As documented by Elliot *et al.* (2007), this is mainly due to the characteristics of the pay setting system, namely highly centralized collective agreements between the central government and trade unions[2]. The fact that civil servants in richer, more productive regions get lower real wages than their peers in more disadvantaged areas may be explained by compensating wage differential, with the public authority paying an additional amount of real income to induce workers to accept a job in a less desirable location. Also, a government may be guided by a redistributive purpose and the fraction of the wage unrelated to productivity and labour market conditions can be considered as either a tax or a subsidy[3]. In addition, the hidden form of this redistributive goal makes it politically more attractive, as Coate and Morris (1995) and Alesina *et al.*(2000) suggest.

The aim of this paper is to investigate the effects of centralization in the public sector wage negotiation on employment, real pays, and inequality. I construct a two-region general equilibrium model in which private and public goods are produced. In

[1] In Italy, Spain, and Germany this is accompanied by a pronounced income disparity between regions (see, respectively, Dell'Arringa *et al.*, 2007, Garcia-Perez and Jimeno, 2007, and Heitmueller and Mavromaras, 2007). Some of these papers look at real wage spatial distributions but, since they use a national price index, their results also apply to nominal pay variations.

[2] In some countries (e.g. France), unilateral decisions taken by the central government are also frequent. See Meurs and Edon (2007).

[3] Alesina *et al.* (2001) find that about half of the public wage bill in the South of Italy can be viewed as a transfer of resources from the North.

each region, the former is sold in a competitive market and is freely tradable, whereas the production of the public, nontradable, good is financed by taxes. Labour markets present search frictions and wages are bargained over by unions of workers and employers. People move from one region to another at exogenous rates. Under these assumptions, the region in which the tradable private sector is more productive compared to the public nontradable one exhibits a higher price of the consumption good for the well-known Harrod-Balassa-Samuelson argument[4].

In this setting, opting for a national bargaining process compared to a regional one in the public sector raises the real pays of civil servants in the poorer region, but it also increases unemployment and reduces private sector real wages. By taking care of civil servants of both regions, public sector unions will negotiate a nominal wage that is lower (resp. higher) than the one that would accrue to civil servants in the richer (resp. poorer) region under a regional bargaining scheme. This enhances the cost of producing the nontradable good - and, in turn, the cost of living - in the poorer region and lowers it in the richer one. Private sector real wages move accordingly. Unemployment in the less productive region is higher under a centralized negotiation because more more expensive wage costs entail a lower share of public jobs that usually have a higher expected duration. The opposite occurs in the other region.

A national bargaining scheme also ends up having negative consequences on inequality. It widens Paying civil servants of the richer (res. poorer) region less (resp. more) than would be implied under a regional bargaining scheme, widens (resp. narrows) the gap between public and private sector wages. As concerns the inequality in the entire country, two mechanisms are at work. On the one hand, under a national bargaining process civil servants earn the same nominal pay irrespective of the region they belong to. On the other hand, the same process jacks up the cost of living for people living in the poorer areas and decreases it for the richer. If the productivity gap between the regions is not too huge, the latter outweighs the former and the ratio between the highest and the lowest earnings in the economy goes up.

The paper is organized as follows. Section 2 and 3 present the basic model. Section 4 analyzes the effects of a common nominal wage on real wages and unemployment. Section 5 studies the effects of a subsidy on public employment. Section 6 investigates

[4]See Obstfeld and Rogoff (1996, chapter 4) for a general presentation.

the inequality results. Sections 7 and 8 respectively present the calibration procedure and the quantitative results. Section 9 concludes.

2 The Basic Model

2.1 Preferences and Technology

I consider a country composed by two regions, say a and b. Regions differ only in terms of private sector productivity, while all the other product and labour market parameters are assumed to be the same. Besides the gain in simplicity, this also allows to isolate more starkly the effects of different public wage policies on the regional disparities that will result from the model.

In each region, two intermediate goods and one final consumption good are produced. As respects the intermediate goods, one is produced in the private sector and can be traded across the regions at a competitive price, the other one is public and not tradable[5]. The consumption good is also sold in a competitive market, but it is not tradable. Its production function takes a CES form:

$$Y_i = \left[Q_{p,i}^{\frac{s-1}{s}} + Q_{g,i}^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} \quad \text{with } i \in \{a, b\}, \tag{1}$$

in which $Q_{p,i}$ and $Q_{g,i}$ respectively denote the intermediate good produced in the private sector and the intermediate good produced in the public sector in region i. The elasticity of substitution s is greater than 1 to allow a situation in which some Q_i are equal to zero.

Let P_i and $P(Q_{p,i})$ be the the prices of the consumption good and the private intermediate good in region i. I consider the tradable good $Q_{p,i}$ as the numeraire for the economy of region i.[6] So its price is normalized to 1 and it is equal across the regions. The final good firm in region i minimizes its cost by taking these prices and the amount of the public good provided by the government $Q_{g,i}$ as given. This leads

[5]Police service, environmental protection, the administration of justice are all examples of goods that cannot be traded.

[6]In solving the model by expressing the prices of nontradables in terms of the tradable, I am following Obstfeld and Rogoff (1996, chapter 4, pages 204 - 228).

to the following F.O.C.:

$$p_i \cdot \left(\frac{Q_{p,i}}{Y_i}\right)^{-\frac{1}{s}} = 1 \qquad \text{with } p_i \equiv \frac{P_i}{P(Q_{p,\,i})} \tag{2}$$

Time is continuous and the model is developed in steady-state. In the entire country there is a measure normalized to 1 of workers that are infinitely-lived and risk-neutral.

Any employed worker in region i produces y_i units of the private intermediate good, with $y_a > y_b > 1$. So, the total amount of private intermediate goods are:

$$Q_{p,\,i} = y_i \cdot E_{p,\,i} \qquad \text{with } i \in \{a,\, b\}, \tag{3}$$

with $E_{p,\,i}$ defining the total level of employment in the private sector of region i. In the public sector, the government decides the optimal amount of public goods to be produced in regions a and b maximizing a well-defined welfare function. As in the private sector, the production function is linear and identical in both regions:

$$Q_{g,\,i} = E_{g,\,i}, \qquad \text{with } i \in \{a,\, b\}, \tag{4}$$

with $E_{g,\,i}$ defining the total level of employment in the private sector of region i.

There are frictions on the labor market. As respect to the labour market flows, I make two important assumptions. First, in each region search is undirected; this means that unemployed workers do not direct job search towards a particular sector and both private and public job vacancies have the same probability of meeting workers, since it is the total number of vacancies that enters the matching function for region i. More precisely, the flow of hires, M_i, is a function of the number of vacancies, V_i and the number of unemployed people, the only job-seekers in the economy (no on-the-job search), U_i, with $i \in \{a,\, b\}$. The matching function is written $M_i = m\,(U_i,\, V_i\,)$. Following most of the literature, I consider a Cobb-Douglas functional form: $M_i = U_i^{\eta} \cdot V_i^{1-\eta}$. Labour market tightness is denoted by $\theta_i \equiv V_i/U_i$. The rate at which vacant jobs become filled is $q(\theta_i) \equiv m\,(U_i,\, V_i\,)\,/V_i, = \theta_i^{-\eta}$. A job-seeker moves into employment at a rate $f(\theta_i) \equiv \theta_i\, q(\theta_i) = \theta_i^{1-\eta}$. At an exogenous rate δ_g a public job is destroyed; the private sector separation rate is denoted by δ_p. Public employment contracts have a longer duration. I assume that $\delta_p > \delta_g$.

The second important assumption concerns migration from one region to the other. At an exogenous rate λ_i, an unemployed worker in region i migrates to region j and starts searching for a job there[7].

Let ϕ_i ($i \in \{a, b\}$) designate the share of public job vacancies among all vacancies in region i. The equality between flows in and out each workers' status leads to the following equations:

$$\begin{aligned} E_{p,i}\,\delta_p &= U_i\,(1-\phi_i)\,f(\theta_i) \\ E_{g,i}\,\delta_g &= U_i\,\phi_i\,f(\theta_i) \qquad \text{with } i \in \{a, b\}, \\ \lambda_a U_a &= \lambda_b U_b \end{aligned} \tag{5}$$

where U_i is the level of unemployment in region i. Since $1 = \sum_{n,i} E_{n,i} + \sum_{n,i} U_{n,i}$, with $n \in \{p, g\}$ and $i \in \{a, b\}$ the equations that determine the level of employment in each region are:

$$\begin{aligned} E_a &= \frac{f(\theta_a)\cdot\left(\frac{\phi_a}{\delta_g} + \frac{1-\phi_a}{\delta_p}\right)}{1 + \frac{\lambda_a}{\lambda_b}\left[1 + f(\theta_b)\cdot\left(\frac{\phi_b}{\delta_g} + \frac{1-\phi_b}{\delta_p}\right)\right] + f(\theta_a)\cdot\left(\frac{\phi_a}{\delta_g} + \frac{1-\phi_a}{\delta_p}\right)} \\ E_b &= \frac{\frac{\lambda_a}{\lambda_b} f(\theta_b)\cdot\left(\frac{\phi_b}{\delta_g} + \frac{1-\phi_b}{\delta_p}\right)}{1 + \frac{\lambda_a}{\lambda_b}\left[1 + f(\theta_b)\cdot\left(\frac{\phi_b}{\delta_g} + \frac{1-\phi_b}{\delta_p}\right)\right] + f(\theta_a)\cdot\left(\frac{\phi_a}{\delta_g} + \frac{1-\phi_a}{\delta_p}\right)} \end{aligned} \tag{6}$$

Using the steady-state labour market flows (5) and equations (6), it possible to determine the fraction of public jobs out of total for each region:

$$\frac{E_{g,i}}{E_i} = \frac{\frac{\phi_i}{\delta_g}}{\frac{\phi_i}{\delta_g} + \frac{1-\phi_i}{\delta_p}}, \qquad \text{with } i \in \{a, b\} \tag{7}$$

Moreover, substituting equations (3), (4), and (7) in the demand function (2), p_i can be written as:

$$p_i = \left[1 + \left(\frac{\phi_i}{y_i(1-\phi_i)}\cdot\frac{\delta_p}{\delta_g}\right)^{\frac{s-1}{s}}\right]^{\frac{1}{1-s}} \qquad \text{with } i \in \{a, b\}. \tag{8}$$

[7]Of course considering an exogenous rate does not allow to investigate the effects of different wage policies on migration. The assumption is done for simplicity reasons.

The price of the consumption good is increasing in y_i and decreasing in ϕ_i. As I proceed I will investigate the general implications of this equation.

Let r be the discount rate common to all agents. As usual in the standard search and matching literature (Pissarides, 2000, chapter 1), I impose the one firm - one job assumption in the private sector. The expected discounted utility of an unemployed worker searching for a job of type $n \in \{g, p\}$ in region $i \in \{a, b\}$, U_i verifies the following Bellman equation:

$$rU_i = f(\theta_i) \left[\phi_i W_{g,i} + (1-\phi) W_{p,i} - U_i \right] \tag{9}$$

The instantaneous utility in unemployment is assumed equal to zero for simplicity, and $W_{g,i}$ (resp. $W_{p,i}$) is the discounted present value of being employed in the public (resp. private) sector in region i.

The Bellman equation for a worker of region $i \in \{a, b\}$ employed in sector $n \in \{g, p\}$ is:

$$rW_{n,i} = \frac{w_{n,i}}{p_i} + \delta_n \cdot (U_i - W_{n,i}), \tag{10}$$

where $w_{n,i}/p_i$ is the real wage in sector n of region i.

The Bellman equation for an active private firm is:

$$rJ_{p,i} = \frac{y_i}{p_i} - \frac{w_{p,i}}{p_i} + \delta_p \left(V_{p,i} - J_{p,i} \right), \qquad \text{with } i \in \{a, b\} \tag{11}$$

A firm-worker pair in region i produces y_i units of the intermediate private good; so the expected discounted utility of a firm that has filled its vacancy, $J_{p,i}$, is the sum of its revenues and the capital loss occurring at rate δ_p.

To post a vacancy, intermediate firms need to import an input good, whose price k is determined in the international market by an infinitely elastic supply[8]. For a convenient normalization, I impose that firms buy 1 unit of an input good per unit of time[9]. So, as long as the job position remains idle, the flow cost for the firm is equal

[8] Advertising costs via Internet may be an example.

[9] In a standard search and matching model the flow cost of a vacancy is expressed in terms of the final consumption good. My assumption that firms need to buy a different input with a fixed price is done for simplicity reasons. If private firms needed a consumption good to hire a worker, their behaviour would differ across regions not only for the productivity gap but also because vacancy costs would increase with the cost of living. This further complication would jeopardize the analytical tractability of the model.

to k/p_i. The expected value of vacancy, $V_{p,\,i}$, is given by the sum of such a cost and the capital gain that accrues from the match, multiplied by the job filling rate:

$$rV_{p,\,i} = -\frac{k}{p_i} + q(\theta_i)\left(J_{p,\,i} - V_{p,\,i}\right), \quad \text{with } i \in \{a,\, b\}. \tag{12}$$

2.2 Tightness and wage determination in the private sectors

As common in search and matching models, free entry and a rent sharing rule determine the equilibrium values of tightness and nominal wage in sector i, θ_i and $w_{p,\,i}$. Free-entry of vacancies implies that the expected value of an unfilled job $V_{p,i}$ must be equal to zero. Substituting this into (11) and (12), one gets the usual vacancy-supply curve:

$$\frac{y_i - w_{p,\,i}}{r + \delta_p} = \frac{k}{q(\theta_i)} \quad \text{with } i \in \{a,\, b\}.$$

The expected discounted revenues are equal to the expected cost of posting a vacancy.

In each region, the private sector wage is negotiated via collective bargaining between unions of firms and unions of workers. Such an assumption seems plausible for many countries in Continental Europe, where individual bargaining is rare and the sectoral level of negotiation often plays a major role. The expected discounted utility of the unions of private sector workers and firms in region i are respectively:

$$rT^{W}_{p,\,i} = \frac{w_{p,\,i}}{p_i} \cdot E_{p,\,i} \tag{13}$$

$$rT^{F}_{p,\,i} = \left(\frac{y_i}{p_i} - \frac{w_{p,\,i}}{p_i}\right) \cdot E_{p,\,i} \tag{14}$$

The underlying assumption is that the trade union behaves in a utilitarian way, caring about the sum of its members' incomes[10]. I consider an axiomatic Nash solution. The nominal wage received by private employees in region i solves the problem:

$$\begin{aligned} w_{p,\,i} \;=\; & \text{argmax}\, \left[T^{W}_{p,\,i} - \bar{T}^{W}_{p,\,i}\right]^{\beta} \left[T^{F}_{p,\,i} - \bar{T}^{F}_{p,\,i}\right]^{1-\beta} \\ \text{s.t.} \quad & T^{W}_{p,\,i} > \bar{T}^{W}_{p,\,i} \\ & T^{F}_{p,\,i} > \bar{T}^{F}_{p,\,i} \quad \text{with } i \in \{a,\, b\}. \end{aligned} \tag{15}$$

[10]Recall that I assumed the instantaneous utility of unemployed workers is equal to zero

The terms $\bar{T}^{W}_{p,i}$ and $\bar{T}^{F}_{p,i}$ denote respectively the threat points for the unions of workers and firms. For simplicity I impose both threats points equal to zero. If no agreement is concluded, the employees in that sector do not work and do not earn any salary. The firm does not produce and does not pay any wage[11]. The constraints imposed in the maximization mean that both parties have always the possibility to abandon the negotiation if this choice makes them better off. As in Rosen (1997) and Hall and Milgrom (2008), they are not binding: no player has an incentive to quit the negotiation. Parameter β denotes the exogenous bargaining power of unions of workers ($0 < \beta < 1$). Computing the F.O.C. and using (13), and the conditions on the threats points yields to $w_{p,i} = \beta \cdot y_i$. Substituting this value of the nominal wage in the vacancy-supply equation, I get:

$$\mathbb{ZP}(\theta_i) \;\equiv\; (1 - \beta)\, y_i - k \cdot \frac{r + \delta_p}{q(\theta_i)} = 0 \qquad \text{with } i \in \{a,\, b\}. \tag{16}$$

This implicit function is denoted $\mathbb{ZP}(\theta_i) = 0$ because of the the zero profit condition that determines the vacancy/unemployment ratio in the private sector. Notice that $\theta_a > \theta_b$ because $y_a > y_b$.

2.3 The governments' objective and budget constraint

The value function of each local government solves the following Bellman equation:

$$\begin{aligned} r\Pi(E_{g,i}) \;&=\; \max_{V_{g,i}} \left[\left(Y_i - \frac{w_{g,i}}{p_i}\, E_{g,i,t} - \frac{k}{p_i}\, V_{g,i} \right) + \frac{d\,\Pi(E_{g,i})}{d\,t} \right] \\ s.t. \quad \frac{d\,E_{g,i}}{d\,t} \;&=\; V_{g,i} \cdot q(\theta_i) - \delta_g \cdot E_{g,i} \qquad \text{with } i \in \{a,\, b\}\,. \end{aligned} \tag{17}$$

When deciding how many public vacancies must be posted, the public authority of region i maximizes total output net of the wage bill and the vacancy costs. Labour market tightness is taken as given. Notice that at this stage there is no link between regions, so we would get the same results if we considered a central government that

[11] Such threats points are similar to those introduced by Rosen (1997) and Hall and Milgrom (2008). The idea is that a disagreement in the negotiation between unions usually implies a delay in the production, strikes, not massive lay-offs or quits. Actually, in the paper of Hall and Milgrom, the delay in the production involves a flow cost for the firm. For simplicity, I impose it equal to zero.

maximizes a value function equal to the sum $\Pi(E_{g,a}) + \Pi(E_{g,b})$. The F.O.C. condition that satisfies (17) is:

$$\frac{d\,\Pi(E_{g,i})}{d\,E_{g,i}} \;=\; \frac{k}{p_i \cdot q(\theta_i)}, \qquad i \in \{a,\, b\}$$

The marginal value of one additional job must be equal to the expected cost of filling a vacancy. In steady-state, $\frac{d\,E_{g,i}}{dt} \;=\; 0$. Applying the envelope theorem to (17) yields:

$$\frac{d\,\Pi(E_{g,i})}{d\,E_{g,i}} \;=\; \frac{1}{p_i} \cdot \frac{p_i \cdot (d\,Y/d\,Q_{g,i}) \;-\; w_{g,i}}{r \,+\, \delta_g}, \qquad i \in \{a,\, b\}$$

Using equations (2), (3), (4), and (7) one gets that $p_i{\cdot}(d\,Y/d\,Q_{g,i}) \;=\; y_i^{1/s}{\cdot}\left(\frac{\phi_i}{1-\phi_i} \cdot \frac{\delta_p}{\delta_g}\right)^{-1/s}$. Putting together the RHS of the two preceding equations, one gets:

$$\mathbb{G}\,(\phi_i,\, \theta_i,\, w_{g,i}) \;\equiv\; \frac{y_i^{\frac{1}{s}} \cdot \left(\frac{\phi_i}{1-\phi_i} \cdot \frac{\delta_p}{\delta_g}\right)^{-\frac{1}{s}} - w_{g,i}}{r \,+\, \delta_g} \;-\; \frac{k}{q(\theta_i)} \;=\; 0 \quad i \in \{a,\, b\} \tag{18}$$

For each region, the implicit function $\mathbb{G}\,(\phi_i,\, \theta_i\, w_{g,i}) \;=\; 0$ determines the fraction of public vacancies ϕ_i conditional on the level of tightness θ_i and on the value of $w_{g,i}$. Since the public good is not sold in a competitive market but freely provided by the government, its production costs are financed via a tax T_i (expressed in terms of the numeraire) levied on the final good firms and equal to their profits.

$$T_i \;=\; p_i{\cdot}Y_i - Q_{p,i} \;=\; p_i{\cdot}E_{g,i}{\cdot}(d\,Y/d\,Q_{g,i}) \;=\; E_{g,i}{\cdot}\; y_i^{\frac{1}{s}} \cdot \left(\frac{\phi_i}{1-\phi_i}\right)^{-\frac{1}{s}} \qquad i \in \{a,\, b\} \tag{19}$$

The second equality comes from the constant returns to scale property of the final good production functions. The amount of tax per public worker is equal to the marginal value of public employment. By substituting this value to equation (18), government's expected profits are equal to zero. By the same token, the free entry condition and equation (19) respectively ensure that neither the intermediate private firms nor the final good sector get positive expected profits or losses in steady-state. A corollary of the budget balanced condition (19) is the absence of fiscal redistribution among regions: in each one taxes must be equal to fiscal revenues. This condition will be relaxed in the third scenario I will present.

3 Two Public Wage Scenarios

To close the model, an assumption on how public wages are determined is needed. In this section, I consider two different scenarios:

1. *Regional Public Wage Bargaining.* The nominal public wage is negotiated at regional level by local public authorities and unions.

2. *Centralized Public Wage Bargaining.* The public wage negotiation takes place at the central level by national unions and the central government.

In both scenarios, the local authorities benefit from a relative autonomy with respect to the central government, since they are free to choose the optimal level of public job vacancies. Moreover, there is no fiscal redistribution between regions, since all taxes collected in one region are entirely used to finance the provision of the public good there. These assumptions will be relaxed in a third scenario, presented in section 4, characterized by a higher extent of fiscal integration.

3.1 Regional Bargaining

The public wage bargaining process takes place a the regional level. As in the private sector, the rent sharing rule has the following form:

$$w_{g,\,i} \;=\; \text{argmax} \left[\frac{w_{g,\,i}}{p_i} \cdot E_{g,\,i} \right]^{\beta} \left[E_{g,\,i} \left(F'(Q_{g,\,i}) - \frac{w_{g,\,i}}{p_i} \right) \;-\; \frac{k}{p_i} V_{g,\,i} \right]^{1-\beta}$$

with $i \in \{a,\, b\}$. The public union's utility is equal to the sum of the instantaneous utilities of its members. The local government considers the social revenues obtained by employing $E_{g,\,i}$ civil servants, net of the wage bill and the vacancy costs. The fall-back position of both parts is assumed equal to zero.

Proceeding as in section 2.2 and using the steady-state equation $V_{g,i} q(\theta_i) \;=\; E_{g,i}\delta_g$, one easily gets an equation for the nominal public wage:

$$w_{g,\,i} \;=\; \beta \left[y_i^{\frac{1}{s}} \cdot \left(\frac{\phi_i}{1-\phi_i} \cdot \frac{\delta_p}{\delta_g} \right)^{-\frac{1}{s}} \;-\; \frac{k \cdot \delta_g}{q(\theta_i)} \right] \;=\; \beta \cdot \frac{r}{r+\delta_p} \cdot y_i \qquad (20)$$

with $i \in \{a, b\}$. The wage is a fraction β of the value of one more employee in the public sector, the first terms inside the square brackets, net of the vacancy costs. The second equality is obtained by using the implicit functions $\mathbb{G}(\phi_i, \theta_i\, w_{g,i}) = 0$ and $\mathbb{ZP}(\theta_i) = 0$.

Substituting (20) into (18) yields:

$$\begin{aligned}(1-\beta)\, y_i^{\frac{1}{s}} \cdot \left(\frac{\phi_i^*}{1-\phi_i^*} \cdot \frac{\delta_p}{\delta_g}\right)^{-\frac{1}{s}} &= k \cdot \frac{r + (1-\beta)\delta_g}{q(\theta_i)} \\ &= (r + (1-\beta)\delta_g) \cdot \frac{(1-\beta) y_a}{r + \delta_p},\end{aligned} \tag{21}$$

with $i \in \{a, b\}$. Henceforth the superscript * denotes the values of the endogenous variables under this regional scenario[12]. The second equality is obtained by using the zero profit condition $\mathbb{ZP}_i(\theta_i) = 0$. Under regional wage bargaining in the public sector, the fraction of public vacancies out of total ϕ_i is uniquely determined by equation (21). It is easy to see that $\phi_a < \phi_b$ because $y_a > y_b$. In region a, the private sector is more productive, firms post more vacancies and crowd out more public jobs: from equation (7), the fraction of public employment out of total is lower in region a than in region b.

In turn, this has an impact on the price of the consumption good. Combining (8) and (21), one gets:

$$p_i^* = \left[1 + \left(\frac{r + (1-\beta)\delta_g}{r + \delta_p}\, y_i\right)^{1-s}\right]^{\frac{1}{1-s}} \quad \text{with } i \in \{a, b\}.$$

Region a exhibits a higher cost of living because $y_a > y_b$. This is the well-known *Harrod-Balassa-Samuelson effect*, that predicts that countries with a higher productivity in tradables compared to nontradables have higher price levels (see Obstfeld and Rogoff, 1996, chapter 4, for a detailed exposition). Because of decreasing marginal returns to labour in the final good production function, a lower share of public jobs raises the marginal value of employment in the public sector, namely 1 multiplied by the tax per unit of public good levied on the final firm, $y_i^{\frac{1}{s}} \cdot \left(\frac{\phi_i^*}{1-\phi_i^*} \cdot \frac{\delta_p}{\delta_g}\right)^{-\frac{1}{s}}$ from equation (19). So, the higher the marginal productivity of labour y_i in the private tradable sector, the

[12] As we will see in the next sections, there is no need to add it for θ_i since tightness does not change with the different scenarios.

higher will be the relative cost - paid via taxes - of the public nontradable good. In turn, this translates into a higher price of the composite consumption good, p_i^{*}[13].

It is not possible to ascertain analytically whether employment is higher in region a than in region b. In the region with a greater private sector productivity, vacancy creation and labour market tightness are higher, but there is also a lower share of public jobs, that enjoy a longer expected duration (since $\delta_g < \delta_p$).

Finally, using $\mathbb{ZP}_i(\theta_i) = 0$, equation (21), and the one determining p_i^*, real wages may be expressed in terms of the exogenous variables:

$$\frac{w_{p,i}^*}{p_i^*} = \beta \left[y_i^{s-1} + \left(\frac{r + (1-\beta)\delta_g}{r + \delta_p} \right)^{1-s} \right]^{\frac{1}{s-1}}$$

$$\frac{w_{g,i}^*}{p_i^*} = \beta \frac{r}{r + \delta_p} \left[y_i^{s-1} + \left(\frac{r + (1-\beta)\delta_g}{r + \delta_p} \right)^{1-s} \right]^{\frac{1}{s-1}} \qquad \text{with } i \in \{a, b\}.$$

Real wages positively depend on y_i. So, $\frac{w_{p,a}^*}{p_a^*} > \frac{w_{p,b}^*}{p_b^*}$ and $\frac{w_{g,a}^*}{p_a^*} > \frac{w_{g,b}^*}{p_b^*}$. Notice also that in each region, public sector wages are lower than private ones[14].

3.2 Centralized Bargaining in the Public Sector

In this second scenario, I assume that the nominal public sector wage is bargained over at the national level. A national union representing all the civil servants in the economy negotiates with the central government. The Nash bargaining problem is:

$$w_g = \operatorname{argmax} \left[\frac{w_g}{p_a} \cdot E_{g,a} + \frac{w_g}{p_b} \cdot E_{g,b} \right]^{\beta} \left[\sum_{i=a,b} E_{g,i} \left(F'(Q_{g,i}) - \frac{w_g}{p_i} \right) - \frac{k}{p_i} V_{g,i} \right]^{1-\beta}$$

[13] Actually, the Harrod-Balassa-Samuelson effect hinges on the assumption of perfect labour mobility across tradable and nontradable sectors, that equalizes wages and the correspondent values of the marginal product. Here the mechanism is different, as undirected search prevents workers from choosing in which sector they want to apply.

[14] It remains to check if $\frac{w_{g,a}^*}{p_a^*}$ is greater than $\frac{w_{p,b}^*}{p_b^*}$. After some algebra, one gets that this is the case if:

$$\left(\frac{r + (1-\beta)\delta_g}{r + \delta_p} \right)^{1-s} \cdot \left[1 - \left(\frac{r}{r + \delta_p} \right)^{s-1} \right] < y_a^{s-1} \cdot \left(\frac{r}{r + \delta_p} \right)^{s-1} - y_b^{s-1}$$

The resulting F.O.C. is equal to:

$$w_g \left(\frac{E_{g,\,a}}{p_a} + \frac{E_{g,\,b}}{p_b} \right) \; = \; \beta \left[\sum_{i\,=\,a,b} \frac{E_{g,\,i}}{p_i} \cdot \left(y_i^{\frac{1}{s}} \cdot \left(\frac{\phi_i}{1-\phi_i} \cdot \frac{\delta_p}{\delta_g} \right)^{-\frac{1}{s}} - \frac{k \cdot \delta_g}{q(\theta_i)} \right) \right]$$

Using the steady state equations in (5) and the implicit function $\mathbb{G}_i(\phi_i,\, \theta_i,\, w_g) \; = \; 0$, the above equation simplifies to:

$$w_g = \gamma \cdot w_{g,\,a} \, + \, (1-\gamma) \cdot w_{g,\,b}, \tag{22}$$

in which

$$\gamma \; \equiv \; \frac{\frac{f(\theta_a)\phi_a}{p_a}}{\frac{f(\theta_a)\phi_a}{p_a} + \frac{\lambda_a}{\lambda_b} \frac{f(\theta_b)\phi_b}{p_b}}$$

and $w_{g,\,a}$ and $w_{g,\,b}$ have the same functional form of the nominal public wages in the first scenario (see equation 20). Under national bargaining, the nominal public sector wage is a weighted average of the wages obtained via regional negotiation. The endogenous weight are γ and $1-\gamma$. Notice that γ positively depends on ϕ_a and θ_a and it is negatively affected by ϕ_b and θ_b. The larger the fraction of public sector vacancies (and in turn, the fraction of public sector jobs) and the labour market tightness in region i, the closer the identical nominal wage w_g is to productivity and tightness of that region. The nominal public wage w_g can also be rewritten by using the zero profit conditions $\mathbb{ZP}(\theta_i) \; = \; 0$ and the equilibrium equations in the public sector $\mathbb{G}(\phi_i,\, \theta_i,\, w_g) \; = \; 0$:

$$w_g \; = \; \beta \cdot \frac{r}{r \, + \, \delta_p} \left[\gamma \cdot y_a \, + \, (1-\gamma) \cdot y_b \right]. \tag{23}$$

Notice that, apart from the exogenous migration flows, w_g is the only link between region a and b, since it depends on $\theta_{g,\,a}$ and $\theta_{g,\,b}$, ϕ_a, and ϕ_b. As in the first scenario, θ_i is uniquely determined by the zero profit equation $\mathbb{ZP}(\theta_i) \; = \; 0$, with $i \in \{a,\, b\}$, while the equilibrium conditions in the public sector $\mathbb{G}(\phi_i,\, \theta_i,\, w_g) \; = \; 0$ with $i \in \{a,\, b\}$ form a system of two equations in the two unknowns, ϕ_a and ϕ_b. Lemma 1 presents the result:

Lemma 1 *If* $y_b \; > \; y_a \cdot \frac{s\beta r}{s\beta r + r + \delta_g (1-\beta)}$, *the system of equations* $\mathbb{G}(\phi_i,\, \theta_i,\, w_g) \; = \; 0$ *with* $i \in \{a,\, b\}$ *admits a unique solution in* ϕ_a *and* ϕ_b.

Substituting the RHS of (22) in $\mathbb{G}(\phi_a,\, \theta_a,\, w_g) \,=\, 0$ and differentiating with respect to ϕ_a and ϕ_b, we get $\frac{d\,\mathbb{G}_a}{d\,\phi_a} < 0$ and $\frac{d\,\mathbb{G}_a}{d\,\phi_b} > 0$. The sign of this last derivative comes from the fact that w_g is negatively affected by ϕ_b via the term γ. Therefore, the implicit function $\mathbb{G}(\phi_a,\, \theta_a,\, w_g) \,=\, 0$ describes an increasing relationship in the $(\phi_b,\, \phi_a)$ space. Similarly, we have $\frac{d\,\mathbb{G}_b}{d\,\phi_a} < 0$, because of the positive effect of ϕ_a on γ. It remains to check the sign of $\frac{d\,\mathbb{G}_b}{d\,\phi_b}$.
Replacing the RHS of (22) in $\mathbb{G}(\phi_b,\, \theta_b,\, w_g) \,=\, 0$ and differentiating with respect to ϕ_b, one gets:

$$\frac{d\,\mathbb{G}_b}{d\,\phi_b} \;=\; \frac{1}{d\,\phi_b}\cdot d\left[y_b^{\frac{1}{s}}\cdot\left(\frac{\phi_b}{1-\phi_b}\cdot\frac{\delta_p}{\delta_g}\right)^{-\frac{1}{s}}\right] - \frac{\beta\cdot r}{r\,+\,\delta_p}\,(y_a - y_b)\,\frac{d\,\gamma}{d\,\phi_b}. \tag{24}$$

After some algebra, it becomes:

$$\frac{d\,\mathbb{G}_b}{d\,\phi_b} = \frac{1}{s}\frac{1}{1-\phi_b}\frac{1}{\phi_b}\cdot\left\{ -\frac{r\,+\,\delta_g(1-\beta)}{r\,+\,\delta_p} y_b \,-\, \left[1-(1-\gamma)F'(Q_{g,\,b})^{1-s}\right]\frac{\beta r}{r\,+\,\delta_p}(y_a - y_b)\gamma \right\} + \\ + \frac{\beta r}{\phi_b(r\,+\,\delta_p)}(y_a - y_b)\gamma(1-\gamma)$$

Notice that $0 < F'(Q_{g,\,b})^{1-s} = \left[1 + \left(\frac{Q_{p,\,b}}{Q_{g,\,b}}\right)^{\frac{s-1}{s}}\right]^{-1} < 1$. Therefore, a sufficient condition for a negative derivative is:

$$\frac{1}{s}\cdot\frac{r\,+\,\delta_g(1-\beta)}{r\,+\,\delta_p}\cdot y_b \;>\; \frac{\beta\cdot r}{r\,+\,\delta_p}(y_a - y_b),$$

or, equivalently, $y_b > y_a \cdot \frac{s\beta r}{s\beta r + r + \delta_g(1-\beta)}$. If such inequality holds, $\frac{d\,\mathbb{G}_b}{d\,\phi_b} < 0$, and the implicit function $\mathbb{G}(\phi_b,\, \theta_b,\, w_g) \,=\, 0$ describes a decreasing relationship in in the $(\phi_b,\, \phi_a)$ space. Its is then easy to see that thee exists only a solution $(\phi_b^{**},\, \phi_a^{**})$ that solves the system $\mathbb{G}(\phi_i,\, \theta_i,\, w_g) \,=\, 0$, with $i \in \{a,\, b\}$. In turn, the solutions $(\phi_b^{**},\, \phi_a^{**})$ (I denote henceforth the equilibrium values in the second scenario with the superscript $**$) together with θ_a and θ_b obtained via the zero profit conditions, uniquely determine all the endogenous variables of the model. ■

As a comparison between the second equality in (20) with the RHS of (23) makes clear, switching to a national public wage bargaining regime entails a redistribution in nominal terms from the rich to the poor region. Civil servants in region a get a lower nominal wage than it would be implied under the regional bargaining scenario. The opposite occurs to public employees in region b. This result chimes well with most empirical findings (e.g. see Borjas, 1986; Alesina, Danninger, and Rostagno, 2001).

A comparison between region a and b broadly delivers the same results illustrated in the previous scenario. Notice first that θ_i and $w_{p,\,i}$ take the same value obtained under regional public wage bargaining, as the zero profit condition $\mathbb{ZP}(\theta_i) = 0$ and the private sector wage negotiation do not change. So we still have $\theta_a > \theta_b$ and $w_{p,\,a} > w_{p,\,b}$. The price of the consumption good in the second scenario is

$$p_i^{**} = \left[1 + \left(\frac{k(r + \delta_g)}{q(\theta_i)} + w_g \right)^{1-s} \right]^{\frac{1}{1-s}} \quad \text{with } i \in \{a, b\}.$$

So $p_a^{**} > p_b^{**}$ since $\theta_a > \theta_b$. The Harrod-Balassa-Samuelson effect applies even in this framework. As in the first scenario, it is not possible to check at the analytical level which region exhibits the higher employment level. As concerns the real wages, in region a workers are paid more in the private sector than in the public one[15]. In region b, simple computations (available on request) show that $w_{p,b} > w_g$ if $y_b \geq y_a \frac{r}{r+\delta_p}$. Contrarily to the regional bargaining scenario, civil servants in region b are better paid than their colleagues in region a, since they receive the same nominal salary but the cost of living is lower in b[16].

4 Regional vs. National Public Wage Bargaining

The two scenarios presented so far differ only in terms of the wage determination in the public sector . In this section, I wonder what are the consequences on employment, prices, and real wages of applying either regime. Proposition 1 summarizes the results.

[15] Compare the expression for w_g in (23) with $w_{p,\,a} = \beta y_a$

[16] The comparison between $w_{p,\,a}/p_a^{**}$ and $w_{p,\,b}/p_b^{**}$ will be made in the next section.

Proposition 1 *By opting for a centralized wage negotiation in the public sector:*

1. *The share of public employment decreases in the less productive region and increases in the more productive one.*

2. *Unemployment and the cost of living soar in the less productive region, while the opposite occurs in the more productive one.*

3. *Public sector real wages go up (respectively, down) in the less (resp. more) productive region. Private sector real wages go up (respectively, down) in the more (resp. less) productive region.*

Computations are in Appendix 1. Consider first the share of public employment, as defined in (7). In region a, the passage from a regional wage negotiation to a centralized one lowers the local government's expected cost, because civil servants are paid less ($w_{g,a} > w_g$), while the job filling rate $q(\theta_a)$ remains unchanged. On the contrary, the cost of a public job in region b soars, as civil servants are paid more under a national negotiation ($w_{g,b} > w_g$). Because of decreasing returns to labour, the share of public employment out of total must be higher in region a and lower in region b to equalize expected marginal costs and revenues (obtained via taxes) in the the public sectors.

As the equations for p_i^{**} and p_i^* make clear, the cost of living increases with the cost of producing the public nontradable good. So, a centralized wage bargaining process lowers the cost of living in region a and raises it in region b.

A larger proportion of public workers in region a also has a positive impact on employment there, since there are more jobs with a higher expected duration. This explains why the level of employment under the second scenario, E_a^{**}, is greater than E_a^*. Conversely, $E_b^{**} < E_b^*$[17].

Under a national bargaining scheme, in the more productive region real wages go up in the private sector because of the cheaper consumption good; however, real pays decrease in the public sector. The uniform nominal retribution w_g also takes into account the (lower) productivity of civil servants in region b. So, the loss at the nominal

[17] Actually, there is another effect at work. E_i is negatively affected by ϕ_j, with $i, j \in \{a, b\}, i \neq j$ because the higher the fraction of public, more secure, jobs in region i, the lower the level of unemployment there and, in turn, the smaller the fraction of them that will move to region j. So, region a (resp. b) (suffers) from a lower (higher) fraction of civil servants in b (a).

level outweighs the reduction in the cost of living. The opposite mechanism is at work in the less productive region, so real wages go down in the private sector and increase in the public one.

5 Third Scenario: Fiscal Integration

So far, I have assumed that each local government autonomously decides how many public vacancies must be opened, provided that the public budget of any region is balanced. It is a condition that may be realistic in a country with a higher extent of fiscal federalism, but that it is difficult to conceive in more integrated states, where some forms of compensation at the central level exist. In this section, I examine a third scenario, in which the central government favours the creation of public jobs in the poorer region by taking resources from the public sector of the richer one[18]. The public budget of the country remains in balance. Then I will evaluate the effects of this policy under both public wage regimes.

To convey this idea, consider a value function for the government of region i that is identical to the one in (17) apart from the term $\tau_i E_{g,i,t}$:

$$r\Pi(E_{g,i}) = \max_{V_{g,i}} \left\{ \left[Y_i - \left[w_{g,i} + \tau_i \cdot (\mathbf{1}_a(i) - \mathbf{1}_b(i)) \right] \frac{E_{g,i,t}}{p_i} - \frac{k}{p_i} V_{g,i} \right] + \frac{d\,\Pi(E_{g,i})}{d\,t} \right\}$$

$$s.t. \quad \frac{d\,E_{g,i}}{d\,t} = V_{g,i} \cdot q(\theta_i) - \delta_g \cdot E_{g,i} \qquad \text{with } i \in \{a,\, b\}\,.$$

The indicator function $\mathbf{1}_a(i)$ (respectively $\mathbf{1}_b(i)$) has the value 1 if $i = a$ and 0 if $i \neq a$ (resp. 1 if $i = b$ and 0 if $i \neq b$). In words, the central government affects the choice of the local authority in region b by giving a transfer equal to τ_b (expressed in terms of the numeraire) for each public job created. To finance this policy, it uses some of the taxes collected in region a, namely a value τ_a for any public job in region a. So, the private intermediate sector is not directly involved in this mechanism.

Proceeding as in section 2.3, I compute the F.O.C of the above problem, apply the

[18]Of course, one could imagine other forms of redistribution. To raise public employment in the poorer region, the central government may tax the private sector in both regions. Here I consider a more hidden approach. Formally, the central government does not levy any taxes to the workers, but it simply devotes more resources to the public sector of region a compared to region b.

envelope theorem, and put the resulting equations together to obtain:

$$
\begin{aligned}
\mathbb{GI}(\phi_a,\, \theta_a,\, w_{g,\,a}) \;&\equiv\; y_a^{\frac{1}{s}} \cdot \left(\frac{\phi_a}{1-\phi_a} \cdot \frac{\delta_p}{\delta_g} \right)^{-\frac{1}{s}} - \tau_a - \frac{k}{q(\theta_a)} \,(\, r \,+\, \delta_g \,) \,-\, w_{g,\,a} \;=\; 0 \\
\mathbb{GI}(\phi_b,\, \theta_b,\, w_{g,\,b}) \;&\equiv\; y_b^{\frac{1}{s}} \cdot \left(\frac{\phi_b}{1-\phi_b} \cdot \frac{\delta_p}{\delta_g} \right)^{-\frac{1}{s}} + \tau_b - \frac{k}{q(\theta_b)} \,(\, r \,+\, \delta_g \,) \,-\, w_{g,\,b} \;=\; 0
\end{aligned}
\tag{25}
$$

I keep the assumption that the final good firms' profits are totally soaked up by taxes: equation (19) still holds. So, combining (19) and (25), one gets the following budget equations:

$$
\begin{aligned}
\frac{1}{p_a}\left[T_a \,-\, \left(w_{g,\,a} + \frac{k\cdot(r+\delta_g)}{q(\theta_a)} \right) E_{g,\,a} \right] \;&=\; \frac{\tau_a}{p_a}\cdot E_{g,\,a} \\
\frac{1}{p_b}\left[T_b \,-\, \left(w_{g,\,b} + \frac{k\cdot(r+\delta_g)}{q(\theta_b)} \right) E_{g,\,b} \right] \;&=\; -\frac{\tau_b}{p_b}\cdot E_{g,\,b}
\end{aligned}
$$

Public sector of region a exhibits a surplus in real terms equal to RHS of the first equation; conversely, the deficit in region b is in the RHS of the second equation. To make the country's budget balanced:

$$
\frac{\tau_a}{p_a}\cdot E_{g,\,a} \;=\; \frac{\tau_b}{p_b}\cdot E_{g,\,b}
$$

So, to have a budget balanced, I impose:

$$
\tau_a \;=\; \frac{p_a}{E_{g,\,a}}\cdot\frac{E_{g,\,b}}{p_b}\cdot\tau_b \;\;=\;\; \frac{\phi_b\, f(\theta_b)}{p_b}\cdot\frac{p_a}{\phi_a\, f(\theta_a)}\cdot\tau_b, \tag{26}
$$

in which the second equality has been obtained via the steady state equations in 5. Notice that when deciding how many vacancies must be opened, the regional authority in a does not consider equation (26) and take τ_a as given. The rationale of this assumption is that, in this scenario, local institutions do not care about the budget, that becomes a constraint at the country level; so they ignore which values the central government assigns to τ_i to avoid a deficit or a surplus in steady-state.

Consider now the effects of this policy on the public wages. Compared to the first two scenarios, the presence of τ_a (resp. τ_b) decreases (increases) the surplus of a match if a wage agreement is reached. However, since both τ_a and τ_b enter the Nash problem in an additive form, they do not affect the functional specification of $w_{g,\,i}$ (in case of regional bargaining) and w_g (in case of national one). Computing the F.O.C of the Nash bargaining problem and rearranging, it is easy to see that the value of the public wage under regional bargaining and fiscal integration is equal to the one in (20).

Under national bargaining, the functional form of the public wage is also identical to the one obtained under the second scenario, (23). However, the value of w_g in this third scenario is not the same as in the second one. This because γ depends on ϕ_i, that in turn is determined by the system of $\mathbb{GI}(\phi_i,\, \theta_i,\, w_g) \;=\; 0 \;\; \forall i \in \{a, b\}$.

Proving the existence and uniqueness of a steady-state equilibrium in $[\theta_i,\, \phi_i]$ for $i \in \{a,\, b\}$ involves the same procedure followed in sections 3.1 and 3.2. Under regional bargaining, each implicit equation $\mathbb{GI}(\phi_i,\, \theta_i,\, w_{g,\,i}) \;=\; 0$ uniquely determines the equilibrium value of ϕ_i, with $i \in \{a,\, b\}$. Under national bargaining, the existence and uniqueness can be proved by the same steps of Lemma 1[19].

Proposition 2 describes the effects of this policy.

Proposition 2 *Consider a central government that uses taxes collected in the richer region to create public jobs in the poorer one. Unemployment and the cost of living decrease (resp. increases) in the poorer (resp. the richer) region.*

Computations are in Appendix 2. The intuition is straightforward. Opening a public vacancy in region a is more expensive, because there is an additional cost τ_a for any job created that is needed to finance public employment in region b. This raises the cost of producing the nontradable public good $Q_{g,\,a}$ and, for decreasing returns to labour, reduces the share of public jobs. The price of the consumption good p_a soars accordingly. A lower share of public employment out of the total one worsens the unemployment rate, as there are less jobs with a higher expected duration.

The opposite occurs in region b, where a subsidy equal to $\tau_b E_{g,\,b}$ lowers the cost of posting public vacancies, reducing the cost of the nontradable good and thereby decreasing the cost of living. Unemployment goes down because there is a larger fraction of public jobs, with a longer expected duration.

[19] An additional condition is required on the magnitude of τ_b. See Appendix 2 for details

6 Inequality

By centralizing the public wage bargaining process, governments may harbour a redistributive purpose. Still, the general equilibrium effects of this policy are not as obvious, since a common w_g affects both public vacancy creation and the costs of living in different parts of the country. I focus on wage inequality both within each region and in the overall country. As concerns the latter, analytical computations allow me to consider only one simple measure of earnings dispersion, namely the ratio between the highest and the lowest wage in the economy. In the numerical section I will also look at other inequality indexes. The following Proposition summarizes the results:

Proposition 3 *A centralized wage bargaining in the public sector raises the inequality of earnings in the more productive region and, if y_b is sufficiently high, lowers it in the less productive region. A centralized wage bargaining also raises the ratio between the highest and the lowest real wage in the country, if y_b is sufficiently high.*

Under a centralized wage negotiation, civil servants in a get a lower (resp. a higher) nominal pay than would be implied under a regional bargaining process (see equations 20 and 23). On the other hand, nominal private wages are not affected by the change in the public sector wage regime. Since in region a we have $w_{p,a} > w_{g,a} > w_g$, a national bargaining process in the public sector widens the gap between public and private sector pays, raising wage inequality. In region b the result is less obvious, as we cannot rule out the hypothesis that, under the second scenario, the nominal public wage is (much) higher than the private one. In general, national bargaining in the public sector lowers wage inequality if $w_{p,b} - w_{g,b} > \max(w_{p,b}\,;\, w_g) - \min(w_{p,b}\,;\, w_g)$. In the case of $w_{p,b} > w_g$, the result comes from the fact that $w_{g,b} < w_g$. In the case of $w_{p,b} < w_g$, one needs to show that:

$$\beta y_b - \beta\frac{r}{r+\delta_p}y_b > \beta\frac{r}{r+\delta_p}\left[\gamma y_a + (1-\gamma)y_b\right] - \beta y_b \Leftrightarrow y_b > \frac{1}{2}\gamma(y_a - y_b)$$

A sufficient condition for this inequality to hold is $y_b > \frac{1}{3}y_a$. If the regional disparities in terms of private productivity are not too great (that is, if y_b is not too small compared to y_a), the increase in the public sector pay entailed by a national bargaining process cannot be so strong to allow civil servants to be paid much more than private employees. Wage inequality in region b decreases.

The effects of a centralized wage setting process in the public sector on the overall level of inequality in the country are more difficult to ascertain. On the one hand, civil servants get the same nominal pay irrespective of the region they belong to. On the other hand, national bargaining raises the cost of living in the poorer region and lowers it in the richer region a (recall that $p^{*}*_{a} < p_{a}^{*}$ and $p^{*}*_{b} > p_{b}^{*}$).

Under regional bargaining, the ratio between the highest wage in the economy is $(w_{p,a} \cdot p_{b}^{*})/(p_{a}^{*} \cdot w_{g,b})$. Under national bargaining, the highest wage is $w_{p,a}/p_{a}^{**}$, while the lowest wage can be either w_{g}/p_{a}^{**} or $w_{p,b}/p_{b}^{**}$. In Appendix 3, I show that such a ratio is always lower under the first scenario than under the second if:

$$\frac{p_{b}^{**}}{p_{a}^{**}} > \frac{r + \delta_{p}}{r} \cdot \frac{p_{b}^{*}}{p_{a}^{*}} \tag{27}$$

Intuitively, if the increase in the ratio of the consumption good prices due to the national bargaining regime is high enough, the highest to lowest wage ratio increases because of the cost of living effects mentioned above. It can be shown that inequality (27) is verified if

$$y_{b} > \frac{r + \delta_{p}}{r + (1-\beta)\delta_{g}} \cdot \left(\frac{r}{\delta_{p}(s-1)}\right)^{\frac{1}{s-1}}$$

Again, when y_{b} is sufficiently high, the differences in real wages between the regions are not too wide and

the increase in the public sector pay in region b due to national bargaining is not as strong

7 Calibration

In the numerical analysis I consider a richer model in which I drop some of the simplifying assumptions of the previous sections. The exogenous separation rate and the workers' bargaining power are now assumed to differ between sectors (not between regions). I also account for differences in the regional labour force. I take the month as unit of time. Data refer to the period 2006-2007 in Italy. Italy is an interesting case study. It exhibits huge economic disparities between the North-Center regions and the South ones, the former being 50% richer in terms of average disposable income per

Calibration			
Variables	Values	Interpretation	Source
L_a	2.05	labour force in a	ISTAT (2008)
L_b	1	labour force in b	ISTAT (2008)
δ_p	0.6%	separation rate in the private sector	ISTAT (2010)
δ_g	0.36%	separation rate in the public sector	MEF (2010)
β_p	0.42	bargaining power in the private sector	$\mathbb{ZP}(\theta_{p,\,b}) = 0$
ϕ_a	0.12	share of public employment in a	MEF (2010); ISTAT (2008)
ϕ_b	0.2	share of public employment in b	MEF (2010); ISTAT (2008)
r	0.0025	discount rate	3% on annual basis.
y_a	4.42	private productivity in region a	$p_a/p_b = 1.15$
y_b	2.72	private productivity in region b	$\mathbb{G}(\phi_b,\, \theta_{g,\,b}) = 0$
k	210.7	flow cost of opening a vacancy	$\mathbb{ZP}(\theta_{p,\,a}) = 0$
γ	0.002	Government's preference for a high w_g	$\mathbb{G}(\phi_a, \theta_{g,\,a}) = 0$
$\theta_{p,\,a}$	0.02	tightness in sector p of region a	$\mathbb{NA}(\theta_{p,\,a}, \theta_{g,\,a}, \theta_{g,\,b}) = 0$
$\theta_{g,\,a}$	7.04	tightness in sector g of region a	$E_a/L_a = 0.965$
$\theta_{p,\,b}$	0.01	tightness in sector p of region b	$\mathbb{NA}(\theta_{p,\,b}, \theta_{g,\,a}, \theta_{g,\,b}) = 0$
$\theta_{g,\,b}$	0.00007	tightness in sector g of region b	$E_b/L_b = 0.89$

Table 1. Calibration procedure

capita (ISTAT, 2011). Moreover, no statistically significant difference emerges in nominal public wages paid in the areas of the country (Alesina *et al.*, 2001; Dell'Arringa *et al.*, 2007). So the second scenario is the baseline model that I will consider for the parametrization.

Results are summarized in Table 1. The discount rate is fixed at 0.0025 (3% on an annual basis). Elasticity s is set equal to 1.75, implying a low level of substitution between public and private goods[20]. The Italian Institute of Statistics (ISTAT, 2008; 2010) provides data on the labour force, the unemployment rate, and the aggregate

[20]If it is not unrealistic to suppose a low degree of substitutability between private and public goods, the case is even stronger for that subset of public goods that are nontradable: police services, justice administration, etc...

separation rate[21]. The separation rate from public employment is calculated by the Finance Ministry (MEF, 2010) and, as expected, it is about 65% lower. The share of civil servants out of total employment is obtained from the same official sources. Matching parameter η and the bargaining power of public workers are fixed equal to 0.5.

Once these labour market parameters are known, I use both zero profit conditions $\mathbb{ZP}(\theta_{p,i}) = 0$ (with $i \in \{a, b\}$) to write k and β_p (workers' bargaining power in the private sector) as a function of $\theta_{p,a}$ and $\theta_{p,b}$. Even $\theta_{g,a}$ and $\theta_{g,b}$ are expressed as a function of $\theta_{p,a}$ and $\theta_{p,b}$ by using the employment equations (6) and the fact that in 2007 the unemployment rate was 3.5% in the North-Center and 11% in the South of Italy. Thus the system of two equations $\mathbb{NA}(\theta_{p,a}, \theta_{g,a}, \theta_{g,b}) = 0$ and $\mathbb{NA}(\theta_{p,b}, \theta_{g,a}, \theta_{g,b}) = 0$ allows to find the two unknowns $\theta_{p,b}$ and $\theta_{p,a}$. Once the latter are found, I derive the calibrated values for k and β_p, that results slightly lower than workers' bargaining power in the public sector. This chimes well with the Italian case, where public sector unions are particularly powerful (see again Dell'Arringa *et al.*, 2007).

Private sector productivities y_a and y_b are obtained via function $\mathbb{G}(\phi_b, \theta_{g,b}) = 0$ and imposing that p_a is about 15% higher than p_b[22]. Finally, γ is obtained via the remaining equilibrium condition $\mathbb{G}(\phi_a, \theta_{g,a}) = 0$. Its calibrated value may seem too small, about 0.002. However, thanks to such small figure the model delivers a public sector premium in the North (i.e. $w_g/w_{p,a}$) of about 5%, in line with the results of the empirical literature (Dell'Arringa *et al.*, 2007).

8 Numerical Results

I compare the second scenario calibrated in the previous section with the first scenario in which $w_{g,a} \neq w_{g,b}$. Numerical results are summarized in Table 2. A richer model

[21] I split the Italian regions as follows. In region a I include the North (Piedmont, Valle d'Aosta, Lombardy, Trentino, Alto Adige - South Tyrol, Veneto, Fiuli-Venezia Giulia, and Liguria) and the Center (Tuscany, Umbria, and Marche). In region b there are Abruzzo, Molise, Campania, Puglia, Basilicata, Calabria, Sicily, and Sardinia. I decided to exclude Lazio from the computations because its high share of public employment is due to the presence of Rome, the capital, in its territory.

[22] See Cannari and Iuzzolino (2009). To the best of my knowledge, this is the only study that attempted to gauge the difference in the cost of living between North and South of Italy.

Comparison between the First and Second Scenario		
Variables	**Region a**	**Region b**
cost of living	-12%	-11.6%
employment	-0.09%	7.3%
real wage in the private sector	12.5%	13.0%
real wage in the public sector	17.2%	-28.9%
public sector wage premium	4.21%	-37.2%
Aggregate Variables		
Employment		3.48%
Average real wage		9.19%
Average public sector real wage		-2.75%
Gini index		-10.83%
Highest to lowest wage in the economy		-7.36%
Wage variance		38.7%

Table 2. Simulation Results. Percentage changes when nominal public wages vary by region.

in which some parameters differ by region or by sector confirms the conclusions of Propositions 1 and 3. If nominal public wages depend on local market conditions, the economy would experience a reduction in the cost of living in both regions and an increase in the real value of private sector salaries. The only type of workers that would be harmed by this change of policy are the public sector employees in the most disadvantaged region, b. The most interesting pieces of information in Table 2 concern the aggregate variables, whose variation we would not be able to ascertain analytically. Employment and the average real wage go up, meaning that the reduction in w_g/p_b and E_a are outweighed by the surge in real payments of the other workers and in E_b.

Even more significant are the inequality results. The increase in aggregate wage inequality presented in Proposition 3 is based only on one indicator, the ratio between the highest and the lowest wage in the economy. Here I also consider the Gini index and the wage variance. Diversifying public wages by region decreases the former by about 11% (from 0.11 to 0.124) while raising the latter by 39%. The ratio between the highest and the lowest wage also decreases by 7%. So the only measure that shows an increase in inequality is the wage variance. This is mainly due to the fact that it does not account the effect on wage dispersion caused by the reduction in the unemployment rate. By lowering the unemployment rate, a segmented public sector wage policy also decreases the number of the poorest people in the economy, so squeezing inequality. This channel is captured by the Gini index but not by the wage variance.

8.1 Subsidizing public employment

Table 3 shows the effects of a subsidy on public employment in region b financed by a tax on public sector in region b. The exercise is performed under both public wage policies. I consider a subsidy τ_b equal to 16% of the marginal value of the public good in region b, $y_b^{\frac{1}{s}} \cdot (\phi_b/(1-\phi_b))^{-\frac{1}{s}}$. Most results are in line with the conclusions of Proposition 2. Whatever the public wage regime adopted by the government, this policy pushes workers in b towards a public sector job (ϕ_b increases by about 20%), while the opposite occurs in region a (ϕ_a is more than 10% lower). Decreasing marginal returns to labour imply lower (resp. higher) productivity in the nontradable sector of region b (resp. a). Price levels move accordingly: living in region b is about 5% cheaper, while the consumption good in region a is 2% more expensive. Recall from section 5 that only ϕ_a and ϕ_b are influenced by this policy. Labour market tightness remains the same in both sectors of both regions. Since nominal wages depend only on tightness, they are unaffected too; the change in the cost of living explains the variation in real wages, that increase in b by 4% at least and go down in a by more than 1.5%. In turn, this explains why this policy lowers inequality, as the subsidy in region a financed by taxes in b makes workers in the more productive region poorer and workers in the less productive region richer. All the inequality measures considered in this exercise decrease substantially[23].

The public wage policy adopted by the Government is key in determining the crowding out effect of this policy. When public sector wages differ by region in nominal terms, the subsidy on public employment has a slight positive effect on overall employment (a 0.01% increase). The 0.11% gain obtained in region b is only partially offset by the 0.04 loss in region a. More in detail, the subsidy boosts $E_{g,b}$ by a 16% and crowds out

[23] The ratio between the highest and the lowest wage does not change when $w_{g,a} = w_{g,b} = w_g$. This is because in this scenario the ratio is given by $(w_{g,b}/p_b) \cdot (p_b/w_{p,b})$. Since nominal wages do not change, the ratio does not change too.

Third Scenario		
Variables (% change)	$\boldsymbol{w_g = w_{g,a} = w_{g,b}}$	$\boldsymbol{w_{g,a} \neq w_{g,b}}$
ϕ_a	−13.4	−10.57
E_a/L_a	−0.06	−0.04
ϕ_b	23.16	18.07
E_b/L_b	−1.43	0.11
p_a	1.67	2.11
$w_{g,a}/p_a$	−1.65	−2.07
$w_{p,a}/p_a$	−1.65	−2.07
p_b	−4.13	−5.03
$w_{g,b}/p_b$	4.31	5.29
$w_{p,b}/p_b$	4.31	5.29
Aggregate employment	−0.48	0.01
Average real wage	0.64	−0.22
Gini index	−1.35	−11.9
Wage variance	−19.3	−33.7
Highest to lowest wage	0	−6.99

Table 3. Third Scenario: Financing 16% of the marginal value of the public good in b through taxes in the public sector in a. Results under both public wage policies.

$E_{p,b}$ that falls by 9.2%. In region a, the tax levied on the public sector decreases $E_{g,a}$ by 10%; workers tilt towards the private sector, so $E_{p,a}$ is 3.7% higher.

9 Conclusions

The poor responsiveness of public sector wages to local market conditions has several implications for unemployment, real wages, and inequality.

A national bargaining scheme has negative effects on the poorer, less productive, regions of the state. Unemployment soars and private sector earnings are squeezed.

Inequality may also increase, casting some doubts on the redistributive purpose that this process is often said to attain.

Finally, the assumption of exogenous labour migration has not allowed to study workers' decisions. Countries in which the cost of living significantly differs among regions do not experience notable flows of internal migration (see Caponi, 2008, for the case of Italy). This can be both the goal and the outcome of such a centralized public wage setting: people from the poor regions get a subsidy not to move to rich areas of the country. Future research on that is needed.

Appendix 1: Comparing the first two scenarios

I first compare ϕ_i^* with ϕ_i^{**} for $i \in \{a, b\}$. Looking at the equilibrium equations in the public sector $\mathbb{G}(\phi_i, \theta_i, w_{g,i}) = 0$ in both scenarios, we have that

$$w_g + \frac{k(r + \delta_g)}{q(\theta_i)} > y_i \frac{r + \delta_g(1-\beta)}{r + \delta_p} \quad \Leftrightarrow \quad \phi_i^* > \phi_i^{**}$$

Simplifying, the first inequality above becomes:

$$w_g > y_i \frac{\beta \cdot r}{r + \delta_p}$$

From equation (23), this inequality never holds (resp. is always respected) when $i = a$ (resp. $i = b$). Therefore, $\phi_a^{**} > \phi_a^*$ and $\phi_b^{**} < \phi_b^*$. In turn, from equation (8), one gets $p_b^{**} > p_b^*$ and $p_a^{**} < p_a^*$.
Now let consider E_i^{**} and E_i^* for $i \in \{a, b\}$. Since θ_i takes the same value in both scenarios, to see if $E_i^{**} > E_i^*$ it is sufficient to look at the derivatives of employment with respect to ϕ_a and ϕ_b. From equation (6) we have:

$$\frac{d\,E_a}{d\,\phi_b} < 0 \quad \text{and} \quad \frac{d\,E_a}{d\,\phi_a} > 0$$

Since $\phi_a^{**} > \phi_a^*$ and $\phi_b^{**} < \phi_b^*$, we have $E_a^{**} > E_a^*$. Similarly:

$$\frac{d\,E_b}{d\,\phi_b} > 0 \quad \text{and} \quad \frac{d\,E_b}{d\,\phi_a} < 0$$

So one gets $E_b^{**} < E_b^*$.

It remains to compare $\frac{w_g}{p_i^{**}}$ with $\frac{w_{g,\,i}}{p_i^*}$:

$$w_g \left\{ 1 + \left[\frac{k}{q(\theta_i)} \, (\, r \, + \, \delta_g \,) \, + \, w_g \right]^{1-s} \right\}^{\frac{1}{s-1}} \; > \; w_{g,\,i} \left\{ 1 + \left[\frac{k}{q(\theta_i)} \, (\, r \, + \, \delta_g \,) \, + \, w_{g,\,i} \right]^{1-s} \right\}^{\frac{1}{s-1}} \quad \Leftrightarrow$$

$$w_g^{s-1} \, + \, \left[\frac{k}{w_g \cdot q(\theta_i)} \, (\, r \, + \, \delta_g \,) \, + \, 1 \right]^{1-s} \; > \; w_{g,\,i}^{s-1} \, + \, \left[\frac{k}{w_{g,\,i} \cdot q(\theta_i)} \, (\, r \, + \, \delta_g \,) \, + \, 1 \right]^{1-s}$$

When $i \, = \, b$, this inequality is always verified, as $w_g \, > \, w_{g,b}$. On the contrary, when $i \, = \, a$, the LHS is always lower than the RHS because $w_g \, < \, w_{g,\,a}$. So, $\frac{w_g}{p_a^{**}} \, < \, \frac{w_{g,\,a}}{p_a^*}$ and $\frac{w_g}{p_b^{**}} \, > \, \frac{w_{g,\,b}}{p_b^*}$.

Appendix 2: Fiscal Integration

Existence under Regional Bargaining

Under regional bargaining, $\mathbb{GI}(\phi_i, \, \theta_i, \, w_{g,\,i}) \, = \, 0$ uniquely determines ϕ_i, for $i \, \in \{a, \, b\}$, because θ_i is found via the zero profit condition $\mathbb{ZP}(\theta_i) \, = \, 0$ and $w_{g,\,i}$ is a function of ϕ_i only.

Existence under National Bargaining

By differentiating the implicit function $\mathbb{GI}(\phi_b, \, \theta_b, \, w_g) \, = \, 0$ and applying the implicit function theorem, it is easy to see that, as under the second scenario, $\frac{d\phi_b}{d\phi_a}\Big|_{\mathbb{GI}_b = 0} \, < \, 0$ if $y_b \, > \, y_a \cdot \frac{s\beta r}{s\beta r + r + \delta_g(1-\beta)}$. So $\mathbb{GI}(\phi_b, \, \theta_b, \, w_g) \, = \, 0$ describes a decreasing relationship in the $(\phi_b, \, \phi_a)$ space.
Let consider $\mathbb{GI}(\phi_a, \, \theta_a, \, w_g) \, = \, 0$:

$$y_a^{\frac{1}{s}} \cdot \left(\frac{\phi_a}{1 - \phi_a} \cdot \frac{\delta_p}{\delta_g} \right)^{-\frac{1}{s}} - \frac{\Gamma_b}{\Gamma_a} \cdot \tau_b - \frac{k}{q(\theta_a)} \, (\, r \, + \, \delta_g \,) \, - \, w_g \; = \; 0$$

in which $\Gamma_i \, \equiv \, f(\theta_i) \frac{\phi_i}{p_i}$ Differentiating with respect to ϕ_a, one gets:

$$\frac{d \, \mathbb{GI}_a}{d \, \phi_a} \; = \; - \frac{1}{s(1 - \phi_a)\phi_a} y_a^{\frac{1}{s}} \cdot \left(\frac{\phi_a}{1 - \phi_a} \cdot \frac{\delta_p}{\delta_g} \right)^{-\frac{1}{s}} \; + \; \Gamma_b \Gamma_a^{-2} {\Gamma'}_a \tau_b \, - \, \frac{d \, w_g}{d \, \phi_a},$$

in which $\Gamma_i' \equiv \frac{d\Gamma_i}{d\phi_i} > 0$. Notice that:

$$\frac{d\,w_g}{d\,\phi_a} = \frac{\beta r}{r + \delta_p}\frac{\Gamma_b \Gamma_a'(y_a - y_b)}{(\Gamma_a + \Gamma_b)^2}$$

So, $\frac{d\,\mathbb{GI}_a}{d\,\phi_a} < 0$ if:

$$\Gamma_b \Gamma_a' \left[\Gamma_a^{-2}\tau_b - \frac{\beta r}{r + \delta_p}\frac{y_a - y_b}{(\Gamma_a + \Gamma_b)^2}\right] < 0 \Leftrightarrow \tau_b < \frac{\beta r}{r + \delta_p}\gamma^2 (y_a - y_b) \tag{28}$$

Notice that

$$\begin{aligned}
\gamma \equiv \frac{\frac{f(\theta_a)\phi_a^{***}}{p_a^{***}}}{\frac{f(\theta_a)\phi_a^{***}}{p_a^{***}} + \frac{\lambda_a}{\lambda_b}\frac{f(\theta_b)\phi_b^{***}}{p_b^{***}}} &= \frac{\Gamma_a}{\Gamma_a + \Gamma_b} \\
> \frac{f(\theta_a)\phi_a^{***}}{f(\theta_a)\phi_a^{***} + \frac{\lambda_a}{\lambda_b}\frac{f(\theta_b)\phi_b^{***}}{p_b^{***}}} & \qquad \text{because } d\,\gamma/d\,p_a < 0 \text{ and } 0 < p_a < 1 \\
> \frac{\phi_a^{***}}{\phi_a^{***} + \frac{\lambda_a}{\lambda_b}\frac{\phi_b^{***}}{p_b^{***}}} & \qquad \text{because } f(\theta_a) > f(\theta_b) \\
> \frac{1}{1 + \frac{1}{p_b^{***}}\frac{\lambda_a}{\lambda_b}} = \frac{p_b^{***}}{p_b^{***} + \frac{\lambda_a}{\lambda_b}} & \qquad \text{because } d\,\gamma/d\,\phi_a > 0 \text{ and } d\,\gamma/d\,\phi_b < 0 \\
> \frac{p_b^*}{p_b^* + \frac{\lambda_a}{\lambda_b}} & \qquad \text{because } p_b^{***} > p_b^*,
\end{aligned}$$

in which the superscript *** denotes the equilibrium value of a variable under the third scenario. To prove that $p_b^{***} > p_b^*$, I consider the equilibrium equations $\mathbb{G}(\phi_b, \theta_b, w_g) = 0$ and $\mathbb{GI}(\phi_b, \theta_b, w_g) = 0$. They can be rewritten in the following way:

$$y_b^{\frac{1}{s}} \cdot \left(\frac{\phi_b^*}{1 - \phi_b^*} \cdot \frac{\delta_p}{\delta_g}\right)^{-\frac{1}{s}} = \frac{k}{q(\theta_b)}\,(r + \delta_g) + \frac{\beta r}{r + \delta_g} y_b$$

$$y_b^{\frac{1}{s}} \cdot \left(\frac{\phi_b^{***}}{1 - \phi_b^{***}} \cdot \frac{\delta_p}{\delta_g}\right)^{-\frac{1}{s}} = \frac{k}{q(\theta_b)}\,(r + \delta_g) + \frac{\beta r}{r + \delta_g} y_b + \gamma \frac{\beta r}{r + \delta_g}(y_a - y_b) - \tau_b$$

It is easy to see that $\phi_b^{***} < \phi_b^*$ if and only if inequality in (28) holds. In turn, for the equation (8), $p_b^{***} > p_b^*$ if and only if $\phi_b^{***} < \phi_b^*$.

Since $p_b^* = \left\{ 1 + \left[\frac{y_b}{r + \delta_p}\,(r + (1 - \beta)\delta_g\,)\right]^{1-s}\right\}^{\frac{1}{1-s}}$, a sufficient condition for inequality in (28) to hold is

$$\tau_b < \frac{\beta r}{r + \delta_p}(y_a - y_b) \cdot \frac{\left\{1 + \left[\frac{y_b}{r+\delta_p}(r + (1-\beta)\delta_g)\right]^{1-s}\right\}^{\frac{1}{1-s}}}{\left\{1 + \left[\frac{y_b}{r+\delta_p}(r + (1-\beta)\delta_g)\right]^{1-s}\right\}^{\frac{1}{1-s}} + \frac{\lambda_a}{\lambda_b}} \tag{29}$$

If inequality in (29) holds, $\frac{d\,\mathbb{GI}_a}{d\,\phi_a} < 0$.
Similarly, we have:

$$\frac{d\,\mathbb{GI}_a}{d\,\phi_b} = \Gamma_b' \left[-\Gamma_a^{-1}\tau_b + \frac{\beta r}{r + \delta_p}\frac{\Gamma_a(y_a - y_b)}{(\Gamma_a + \Gamma_b)^2}\right] > 0 \Leftrightarrow \tau_b < \frac{\beta r}{r + \delta_p}\gamma^2(y_a - y_b)$$

Again, a sufficient condition for the last inequality is the one in (29).
To conclude, if the condition in (29) holds, $\frac{d\,\mathbb{GI}_a}{d\,\phi_a} < 0$ and $\frac{d\,\mathbb{GI}_a}{d\,\phi_b} > 0$, and the implicit function $\mathbb{GI}_a(\phi_a, \theta_a, w_g) = 0$ describes an increasing relationship in the (ϕ_b, ϕ_a) space. It is then easy to check that there exists a unique vector $(\phi_b^{***}, \phi_a^{***})$ that solves the system composed by $\mathbb{GI}_a(\phi_a, \theta_a, w_g) = 0$ and $\mathbb{GI}_b(\phi_b, \theta_b, w_g) = 0$.

Comparative statics

Under regional bargaining, the zero profit condition $\mathbb{ZP}(\theta_i) = 0$ and $\mathbb{GI}_i(\phi_i, \theta_i, w_{g,b}) = 0$ respectively determine θ_i and ϕ_i, for $i \in \{a, b\}$. Both θ_a and θ_b do not change under this third scenario, since $\mathbb{ZP}(\theta_i) = 0$ is unaffected by τ_b, for $i \in \{a, b\}$. From the inspection of $\mathbb{GI}_i(\phi_i, \theta_i, w_{g,i}) = 0$, one gets that τ_b lowers ϕ_a and raises ϕ_b. So we have $\phi_b^{***} > \phi_b^{**}$ and $\phi_a^{***} < \phi_a^{**}$. The remaining algebra is straightforward. Since $\phi_b^{***} > \phi_b^{**}$, then $p_b^{***} < p_b^{**}$ for equation (8) and, consequently, $w_{p,b}/p_b^{***} > w_{p,b}/p_b^{**}$. Moreover, $E_b^{***} > E_b^{**}$ because θ_b takes the same value in both scenarios and E_b increases with ϕ_b.
By the same token, $\phi_a^{***} < \phi_a^{**}$ implies that $p_a^{***} > p_a^{**}$ and $w_{p,a}/p_a^{***} < w_{p,a}/p_a^{**}$. Moreover, $E_a^{***} < E_a^{**}$ because θ_a takes the same value in both scenarios and E_a increases with ϕ_a.
Even under national bargaining one has $\frac{d\,\mathbb{GI}_a}{d\,\tau_b} < 0$ and $\frac{d\,\mathbb{GI}_b}{d\,\tau_b} > 0$. Applying the implicit function theorem to the system composed by $\mathbb{GI}_a(\phi_a, \theta_a, w_g) = 0$ and $\mathbb{GI}_b(\phi_b, \theta_b, w_g) = 0$, I get:

$$\frac{d\,\phi_b}{d\,\tau_b} = -\frac{\frac{d\,\mathbb{GI}_a}{d\,\phi_a} \cdot \frac{d\,\mathbb{GI}_b}{d\,\tau_b} - \frac{d\,\mathbb{GI}_a}{d\,\tau_b} \cdot \frac{d\,\mathbb{GI}_b}{d\,\phi_a}}{\frac{d\,\mathbb{GI}_a}{d\,\phi_a} \cdot \frac{d\,\mathbb{GI}_b}{d\,\phi_b} - \frac{d\,\mathbb{GI}_a}{d\,\phi_b} \cdot \frac{d\,\mathbb{GI}_b}{d\,\phi_a}} > 0.$$

$$\frac{d\,\phi_a}{d\,\tau_b} \;=\; -\frac{\frac{d\,\mathbb{GI}_a}{d\,\tau_b}\cdot\frac{d\,\mathbb{GI}_b}{d\,\phi_b} \;-\; \frac{d\,\mathbb{GI}_a}{d\,\phi_b}\cdot\frac{d\,\mathbb{GI}_b}{d\,\tau_b}}{\frac{d\,\mathbb{GI}_a}{d\,\phi_a}\cdot\frac{d\,\mathbb{GI}_b}{d\,\phi_b} \;-\; \frac{d\,\mathbb{GI}_a}{d\,\phi_b}\cdot\frac{d\,\mathbb{GI}_b}{d\,\phi_a}}$$

Thus we have:

$$\text{sign}\left[\frac{d\,\phi_a}{d\,\tau_b}\right] \;=\; \text{sign}\left[-\frac{d\,\mathbb{GI}_a}{d\,\tau_b}\cdot\frac{d\,\mathbb{GI}_b}{d\,\phi_b} \;+\; \frac{d\,\mathbb{GI}_a}{d\,\phi_b}\cdot\frac{d\,\mathbb{GI}_b}{d\,\tau_b}\right] \tag{30}$$

The RHS in (30) is equal to:

$$\begin{aligned}
&\left\{\frac{\Gamma_b}{\Gamma_a}\left[\frac{d\,F'(Q_{g,b})p_b}{d\,\phi_b} \;-\; \frac{\beta\,r}{r\,+\,\delta_p}(y_a\,-\,y_b)\frac{d\,\gamma}{d\,\phi_b}\right] \;-\; \frac{\beta\,r}{r\,+\,\delta_p}(y_a\,-\,y_b)\frac{d\,\gamma}{d\,\phi_b}\right\} \;=\\
&=\;\frac{\Gamma_b}{\Gamma_a}\frac{d\,F'(Q_{g,b})p_b}{d\,\phi_b} \;-\; \frac{\Gamma_a\,+\,\Gamma_b}{\Gamma_a}\frac{\beta\,r}{r\,+\,\delta_p}(y_a\,-\,y_b)\frac{d\,\gamma}{d\,\phi_b} \;=\\
&=\;\frac{1-\gamma}{\gamma}\cdot\frac{1}{d\,\phi_b}\cdot d\left[y_b^{\frac{1}{s}}\cdot\left(\frac{\phi_b}{1-\phi_b}\cdot\frac{\delta_p}{\delta_g}\right)^{-\frac{1}{s}}\right] \;-\; \gamma\frac{\beta\,r}{r\,+\,\delta_p}(y_a\,-\,y_b)\frac{d\,\gamma}{d\,\phi_b}
\end{aligned}$$

Notice that the last term is negative if it is negative the derivative in (24). So the condition imposed in Lemma 1 to ensure the existence of a unique equilibrium, namely

$$y_b \;>\; s\frac{\beta r}{r\,+\,\delta_g(1-\beta)}(y_a\,-\,y_b),$$

also guarantees that $\frac{d\,\phi_a}{d\,\tau_b} \;<\; 0$.
To sum up the results of the comparative statics, we have that $\phi_b^{***} > \phi_b^{**}$ and $\phi_a^{***} < \phi_a^{**}$. This are exactly the same conclusions reached under regional bargaining. The effects of τ_b on the other variables of interest are therefore identical to that setting.

Appendix 3: Inequality Results

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